Glossary

Convexity

Convexity is a function of the second derivative of a bond's market value with respect to its yield, as shown in the box below.

$$C = \frac{1}{V}\frac{\partial^2 V}{\partial y^2} = \frac{\dfrac{n(n+1)P}{(1+y)^{n+2}} + c\sum_{t=1}^{m\cdot n}\dfrac{t}{m}\dfrac{t+1}{m}\left(1+\dfrac{y}{m}\right)^{-(t+2)/m}}{V}$$	c := coupon amount C := convexity m := # of coupons per year n := years to maturity V := the bond's market value y := yield to maturity

Current Yield

Current yield is the ratio of a year's worth of coupons to the current market price of the bond, expressed as a percentage.

Day Counting

30/360

In the United States, the 30/360 day counting method is used for corporate bonds, U.S. Agency bonds, and mortgage backed securities. The daily interest rate is the quoted annual rate divided by 360. The 30/360 day counting scheme was invented in the days before computers to make the computations easier. For the purposes of computation, all months have 30 days, and all years have 360 days, so tables could be used to look up the value of accrued interest. Using the 30/360 day count, the number of days between *February 25* and *March 5* is ten days, regardless of whether it's a leap year or not. All months are 30 days long, so there will always be ten days between the 25th day of *any* month and the 5th day of the next month.

Actual/360

The Actual/360 day counting method is used for bank deposits and to calculate rates pegged to indices, e.g. LIBOR. The daily interest rate is 1/360 times the quoted annual rate. The number days per month is based on the actual number of calendar days during a given month (28, 29, 30 or 31). The number of days between *February 25* and *March 5* will be five in most years, and six in leap years.

Actual/Actual

Actual/Actual day counting is used for Treasury bonds and notes. The Actual/Actual day counting method is the most intuitive of the day counting schemes. Each year with 365 days has a daily rate equal to 1/365 times the quoted annual rate, while leap years have a daily rate equal to 1/366 times the quoted annual rate. The number of days per month is based on the actual number of calendar days in each given month (28, 29, 30 or 31). The number of days between *February 25* and *March 5* will be five in most years, and six in leap years.

Macaulay Duration

Macaulay duration was developed as a measure which puts coupon and non-coupon bonds on a comparable footing. Recall that all income for a zero-coupon bond is paid on the maturity date and the Macaulay duration of a zero-coupon bond is the number of years until it reaches maturity. Conversely, coupon bonds produce income prior to maturity. In order to compare zero-coupon and coupon bonds, Frederick Macaulay proposed accumulating the weighted (by time to maturity) future value (at maturity) of each coupon and the similarly weighted par value, thereby allowing a coupon bond to masquerade as a zero-coupon bond.

I believe that R. J. Donohue's An Introduction to Cashflow Analysis[1] is one source responsible for fostering the occasional misconception that Macaulay duration is negative. Donohue starts with first derivative of bond value with respect to yield and works backward ultimately finding that Macaulay duration is negative. I beg to differ. Macaulay duration is a (scaled) time weighted sum of discounted coupons and par value,

$$D_{Mac} = \frac{1}{V}\left[\frac{c}{1+r_p} + \frac{2c}{\left(1+r_p\right)^2} + \frac{3c}{\left(1+r_p\right)^3} + \ldots + \frac{mnc}{\left(1+r_p\right)^{mn}} + \frac{mnP}{\left(1+r_p\right)^{mn}}\right]$$

with periodic yield $r_p = \frac{y}{m}$, where *m* represents the number of coupons per year. There simply is no way that such a forward looking, time weighted sum can produce a negative value. Modified duration merely scales Macaulay duration by the periodic yield, so

$$D_{Mod} = \frac{D_{Mac}}{\left(1+r_p\right)}$$

is also positive. Admittedly, the first derivative of bond value with respect to yield *is* negative. When you take the derivative of bond value,

$$V = \frac{c}{1+r_p} + \frac{c}{\left(1+r_p\right)^2} + \frac{c}{\left(1+r_p\right)^3} + \ldots + \frac{mnc}{\left(1+r_p\right)^{mn}} + \frac{mnP}{\left(1+r_p\right)^{mn}}$$, you get

$$\frac{dV}{dr_p} = \frac{-c}{\left(1+r_p\right)^2} + \frac{-2c}{\left(1+r_p\right)^3} + \frac{-3c}{\left(1+r_p\right)^4} + \ldots + \frac{-mnc}{\left(1+r_p\right)^{mn+1}} + \frac{-mnP}{\left(1+r_p\right)^{mn+1}}$$, or

[1] http://www.regentschoolpress.com/BondDuration.pdf provides a relevant excerpt.

$$\frac{dV}{dr_p} = \frac{-1}{1+r_p}\left[\frac{c}{1+r_p} + \frac{2c}{\left(1+r_p\right)^2} + \frac{3c}{\left(1+r_p\right)^3} + \ldots + \frac{mnc}{\left(1+r_p\right)^{m \cdot n}} + \frac{mnP}{\left(1+r_p\right)^{m \cdot n}}\right].$$

The quantity inside the brackets is $V \cdot D_{Mac}$, so $\frac{dV}{dr_p} = \frac{-V \cdot D_{Mac}}{1+r_p} = -V \cdot D_{Mod}$. In summary,

$D_{Mac} = -\left(1+\frac{y}{m}\right)\frac{1}{V}\frac{dV}{dr}$ $= \frac{\frac{n \cdot P}{(1+y)^n} + c\sum_{t=1}^{m \cdot n}\frac{t}{m}\cdot\left(1+\frac{y}{m}\right)^{-t/m}}{V}$	c := coupon amount m := # of coupons per year n := # of years to maturity P := par value V := the bond's market value y := yield to maturity

Modified Duration

Modified duration essentially scales the Macaulay duration by its periodic yield. The practical application of modified duration is as a first order approximation of how the bond price will change as a function of its yield. In practice, you'd use $\Delta V = -\frac{D_{Mod} V \Delta y}{m}$ to find the effect a yield change would have on the bond's market value, but for accurate bond valuation, you should also consider convexity.

$D_{Mod} = \frac{D_{Mac}}{\left(1+\frac{y}{m}\right)} = -\frac{1}{V}\frac{dV}{dr_p}$ $\Delta V = \frac{-D_{Mod} V \Delta y}{m}$	D_{Mac} := Macaulay duration $r_p = \frac{y}{m}$:= the periodic yield m := number of coupons per year V := the bond's market value y := yield-to-maturity

High duration positions (or portfolios) experience greater risk of loss than low duration positions do. Without considering the effect of convexity, a bond with a modified duration of four years will decline in value by four percent for a percentage point increase in interest rates. Analogously, longer duration bonds have increased risk of loss due to rate increases. See also: Convexity.

Yield to Maturity

Yield to maturity is defined as the rate of interest that, when used to discount a bond's cash flows, generates the bond's current market price, i.e. it satisfies the following equation:

$$V = \frac{P}{(1+y)^n} + c\sum_{t=1}^{m \cdot n}\left(1+\frac{y}{m}\right)^{-t/m}$$	c := the coupon amount m := number of coupons per year n := years to maturity P := the par value V : = the bond's market value y := yield to maturity

This expression can be solved iteratively to find the yield to maturity, given a bond value.